Chapter 4 Magnetic Circuit

Chapter Outline

4.1 Magnetic Materials

In previous chapter, we knew that magnetic field can be induced by the free charges that flow in a current-carrying wire loop and the direction of the induced magnetic field is described by the right-hand rule. On the atomic scale, all materials contain spinning electrons that circulate in orbits, and these electrons can also produce magnetic fields if each of theirs magnetic moments is properly oriented. Thus, a resultant magnetic moment in a macroscopic substance can be observed and such a substance is then said to be *magnetised* and this type of substance is called magnetic material.

A magnetic material is said to be *linear*, *isotropic*, or *homogenous* if it magnetic properties (i.e. μ_r and χ_m) is linear over a specified range of field, independent of the direction of field, or does not vary through out the whole medium of the material, respectively. Magnetic materials also classified as soft and hard materials. Soft materials are normally used as the magnetic core materials for inductors, transformers, and actuators in which the magnetic fields vary frequently. Hard materials or sometime called as permanent magnets are used to generate static magnetic fields in electric motors.

The magnetisation in a material substance is associated with atomic current loops generated by two principal mechanisms: (1) orbital motions of the electrons around the nucleus and similar motions of the protons around each other in the nucleus and (2) spinning motions of the electrons around its own axis. The magnetic moment of an electron is due to the combination of its orbital motion around nucleus and spinning motions around its own axis. Similarly, the magnetic moment of the nucleus also consist of the orbital and spin magnetic moments, which are much smaller than that of the electron. This is because the mass of the nucleus is larger than the mass of electron. Thus, the total magnetic moment of an atom is usually assumed to be calculated by the vector sum of the magnetic dipole moments of its electrons.

If $\overline{\mathbf{m}}$ is the *average magnetic dipole moment per atom*, and if N is the number of atoms per unit volume, the magnetisation per unit volume, **M** is defined as

$$\mathbf{M} = N\overline{\mathbf{m}}\,. \tag{4.1}$$

The unit of **M** is given as amperes/meter.

The magnitude of the individual magnetic moment m of a loop area A is calculated as

$$m = \text{current } I \times \text{loop area } A. \tag{4.2}$$

The direction of **m** is normal to the plane of the loop in accordance with the right-hand rule.

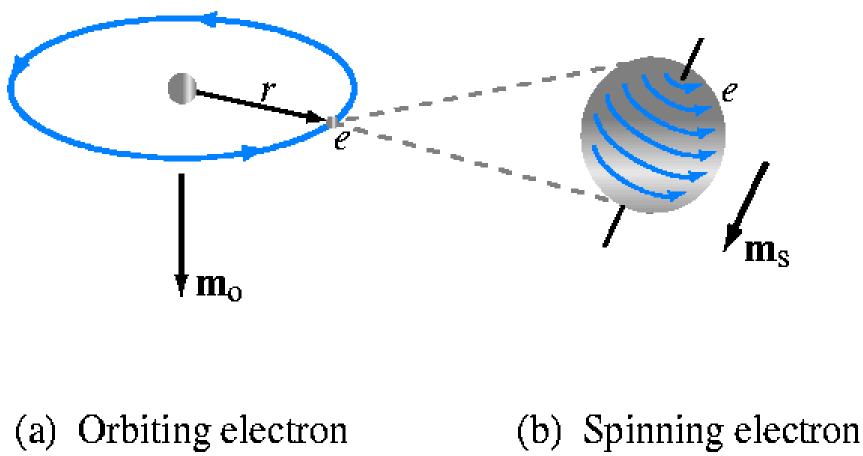


Figure 4.1: The (a) orbital magnetic moment $\mathbf{m_0}$ and (b) spin magnetic moment $\mathbf{m_s}$ of an electron.

The orbital magnetic moment $\mathbf{m_0}$ of an electron can be calculated using the classical model of atom. An electron with charge of $-e$ moving with a constant velocity u in a circular orbit of radius r [figure 4.1(a)] completes one revolution in time $T = 2\pi r/u$. This circular motion of the electron constitutes a tiny current loop with current I given by

$$I = -\frac{e}{T} = -\frac{eu}{2\pi r}\,. \tag{4.3}$$

Thus, the magnitude of the orbital magnetic moment is

$$\begin{aligned} m_0 &= IA \\ &= \left(-\frac{eu}{2\pi r}\right)\left(\pi r^2\right) \\ &= -\frac{eur}{2} \\ m_0 &= -\left(\frac{e}{2m_e}\right)L_e\,, \end{aligned} \tag{4.4}$$

where $L_e = m_e ur$ is the angular momentum of the electron and m_e is its mass. The value of L_e is quantised and is some integer multiple of $\hbar = h/2\pi$ ($L_e = 0, \hbar, 2\hbar, ...$), where h is Plank's constant. Hence, the smallest nonzero magnitude of the orbital magnetic moment of an electron is

$$m_0 = -\frac{e\hbar}{2m_e}. \tag{4.5}$$

In addition to the orbital magnetic moment, the spinning motion of an electron about its own axis produces the spin magnetic moment $\mathbf{m_s}$ and its magnitude is given by the quantum mechanics as

$$m_s = -\frac{e\hbar}{2m_e}, \tag{4.6}$$

which is equal to the minimum orbital magnetic moment m_0. Electrons of an atom with an even number of electrons usually exist in pairs, with the members of a pair having opposite spin directions, thereby cancelling each other's spin magnetic moments. If the number of electrons is odd, the atom will have a nonzero spin magnetic moment due to its unpaired electron.

Relationship of magnetisation vector **M**, magnetic flux density **B**, and permeability μ

The magnetic flux density corresponding to **M** is $\mathbf{B}_m = \mu_0\mathbf{M}$. In the presence of an applied magnetic field **H**, the total magnetic flux density in the magnetic material is given as

$$\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M} = \mu_0(\mathbf{H} + \mathbf{M}), \tag{4.7}$$

where the first term represents the contribution of the external field and the second term represents the contribution of the magnetisation of the material. For diamagnetic and paramagnetic materials at a given temperature, the magnetisation can be written as

$$\mathbf{M} = \chi_m\mathbf{H}, \tag{4.8}$$

where χ_m is a dimensionless quantity called the *magnetic susceptibility* of the material. However, for ferromagnetic material, Eq. (4.8) is nonlinear and also depends on the "history" of the material. Substitute Eq. (4.8) into Eq. (4.7), one gets

$$\mathbf{B} = \mu_0(\mathbf{H} + \chi_m\mathbf{H}) = \mu_0(1 + \chi_m)\mathbf{H}, \tag{4.9}$$

or

$$\mathbf{B} = \mu\mathbf{H}, \tag{4.10}$$

where

$$\mu = \mu_0 (1 + \chi_m), \tag{4.11}$$

is the *magnetic permeability* of the material and its unit is H/m. The relative permeability is defined as

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m, \tag{4.12}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of free space.

The relative permeability and the magnetic behaviour of a material can be used as the basis guideline for classifying materials as *diamagnetic*, *paramagnetic*, or *ferromagnetic*.

4.4.1 Diamagnetism

When a diamagnetic material is exposed to a strong magnetic field, electrons in this material rearrange their orbits (orbital motion of electrons) and creating small persistent currents which oppose the external magnetic field. Hence, diamagnetic materials have a very weak and negative susceptibility to external magnetic fields, and also slightly repelled by the magnetic field. In addition, diamagnetic material does not retain the magnetic properties when the external field is removed and does not have permanent net magnetic moment per atom since its orbital are fully-filled. For example, Neon ($1s^2 2s^2 2p^6$) has 10 electrons in an atom, thus all electrons are paired-up and do not have permanent magnet as shown in the orbital diagram.

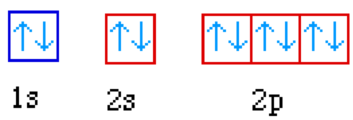


Figure 4.2: Orbital diagram for Neon.

Most elements in the periodic table, including copper, silver, gold, mercury, bismuth, and carbon graphite are diamagnetic. Bismuth and carbon graphite are the strongest diamagnetic materials and they are about eight times stronger than mercury and silver. Other weaker diamagnetic materials include water, diamonds, wood and living tissue. Note that the last three items are carbon-based materials. Some materials behave like superconductor at very low temperature and they are perfect diamagnetic materials which have χ_m = -1 or μ_r = 0 and **B** = 0 (no magnetic field could be established inside superconductor materials since they expel all the magnetic fields that applied on it). The most popular application of diamagnetic materials is magnetic levitation, where an object can be made to float in the air above a strong magnet.

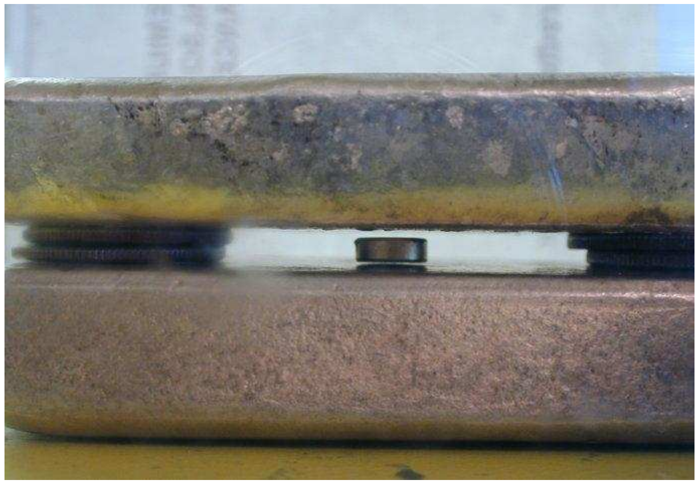


Figure 4.3(a): Diamagnetic levitation using silver.

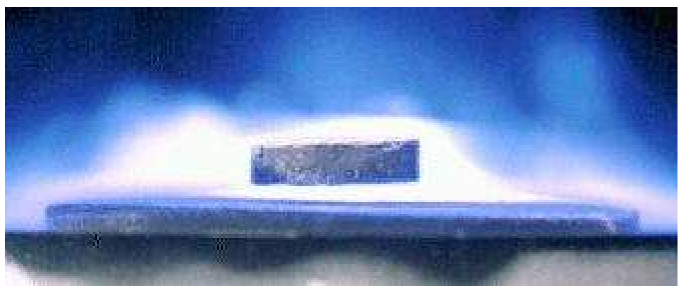


Figure 4.3(b): A superconductor is floating in the air by magnetic levitation.

4.4.2 Paramagnetism

Paramagnetic materials are weakly attracted to magnets and have a small positive susceptibility to magnetic fields. Paramagnetic properties are due to the presence of some unpaired electrons that produce the net spin magnetic moments which tend to align themselves in the direction of the external magnetic field. They do not retain the magnetic properties when the external field is removed. For example, aluminium ($1s^2 2s^2 2p^6 3s^2 3p^1$) has 13 electrons in an atom, thus the unpaired electrons produce net spin magnetic moments and have weak permanent magnetic moment as shown in the orbital diagram.

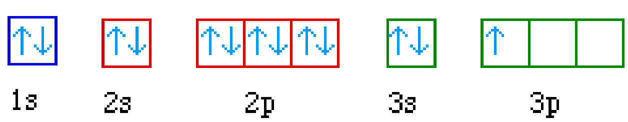


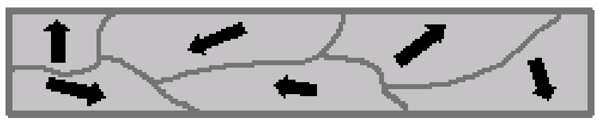
Figure 4.4: Orbital diagram for Aluminium.

Paramagnetic materials are sensitive to temperature and materials like aluminium, uranium and platinum become more magnetic when theirs temperature reduce. Other paramagnetic materials include magnesium, titanium, tungsten, molybdenum, and lithium. Paramagnetic materials are typically considered nonmagnetic since they have very small positive susceptibility (of the order 10^{-5}) as compared to ferromagnetic materials.

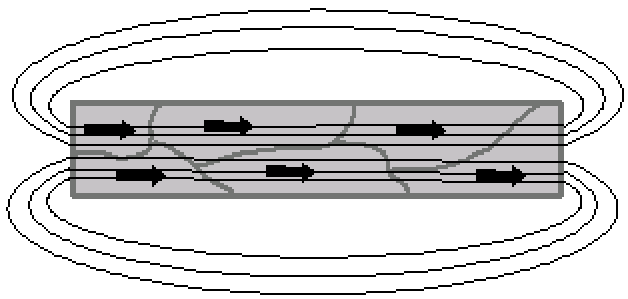
4.4.3 Ferromagnetism

Ferromagnetic materials have a large and positive susceptibility to an external magnetic field. They exhibit a strong attraction to magnetic fields and are able to retain their magnetic properties after the external field has been removed. Iron, nickel, and cobalt are examples of ferromagnetic materials and usually used to fabricate permanent magnets due to the ability to retain their magnetism properties for long time.

Ferromagnetic materials have some unpaired electrons so their atoms have a net magnetic moment. They get their strong magnetic properties due to the presence of magnetic domains within which the magnetic moments of all its atoms (10^{19}) are aligned parallel to each other.



(a) Unmagnetised domains.



(b)Magnetised domains.

Figure 4.5: Comparison of (a) unmagnetised and (b) magnetised domains in a ferromagnetic material.

When a ferromagnetic material is in the unmagnetised state, the domains are nearly randomly organised as shown in figure 4.5(a) and the net magnetic field for the part as a whole is zero. When a magnetising force is applied, the domains become aligned as shown in figure 4.5(b) to produce a strong magnetic field within the part. The boundaries between the adjacent domains consist of thin transition regions called as domain walls.

In addition, strong magnetic ferromagnetic materials like nickel or steel lose all their magnetic properties if they are heated to a critical temperature. This is because the magnetised domains will organise themselves randomly after theirs atoms are being heated. The temperature at which a ferromagnetic material loses its magnetism is called the Curie temperature and it is different for every metal. For example, the Curie temperature for nickel is about 350°C. Table 4.1 summarises the characteristics of magnetic materials.

Table 4.1: Characteristics of magnetic materials.

Characteristic \ Type	**Diamagnetism**	**Paramagnetism**	**Ferromagnetism**
Susceptibility, χ_m	$\chi_m < 0$ (-10^{-5})	$\chi_m > 0$ (10^{-5})	$\chi_m >> 0$
Relative permeability, μ_r	$\mu_r \leq 1$	$\mu_r \geq 1$	$\mu_r >> 1$
Permanent magnetic dipole moment	No ($\mathbf{m_o} + \mathbf{m_s} = 0$)	Yes, but weak ($\mathbf{m_o} + \mathbf{m_s}$ = weak)	Yes, and strong $\lvert\mathbf{m_s}\rvert >> \lvert\mathbf{m_o}\rvert$
Primary magnetisation mechanism	Orbital magnetic moment, $\mathbf{m_o}$	Spin magnetic moment, $\mathbf{m_s}$	Magnetised domain
Direction of **B** relative to **H**	Opposite	Same	Hysteresis loop.
Materials	Bismuth, copper, diamond, gold, lead, mercury, silver, silicon.	Aluminium, magnesium, calcium, chromium, platinum, tungsten, niobium.	Iron, nickel, cobalt.

4.2 Hysteresis

The magnetisation behaviour of the ferromagnetic materials is described by the *B-H* magnetisation curve (hysteresis loop) as shown in figure 4.6.

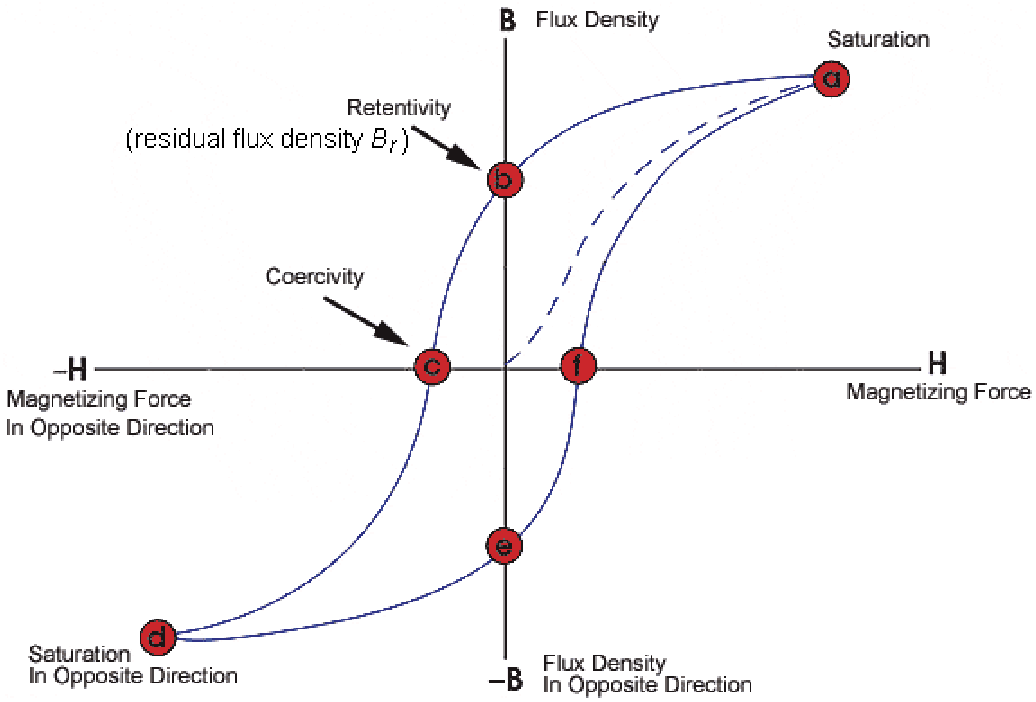


Figure 4.6: Hyteresis loop.

The loop is generated by measuring the magnetic flux **B** of a ferromagnetic material while the magnetising force **H** is changed. A ferromagnetic material that has never been previously magnetised or has been thoroughly demagnetised will follow the dashed line as **H** is increased. As the line demonstrates, the greater the amount of current applied **(H+)**, the stronger the magnetic field in the component **(B+)**. At point “a” almost all of the magnetic domains are aligned and an additional increase in the magnetising force will produce very little increase in magnetic flux. The material has reached the point of magnetic saturation. When **H** is reduced down to zero, the curve will move from point "a" to point “b”. At this point, it can be seen that some magnetic flux remains in the material even though the magnetising force is zero. This is referred to as the point of retentivity on the graph and indicates the remanence or level of residual magnetism in the material. (Some of the magnetic domains remain aligned but some have lost there alignment.) As the magnetising force is reversed, the curve moves to point “c”, where the flux has been reduced to zero. This is called the point of coercivity on the curve. (The reversed magnetising force has flipped enough of the domains so that the net flux within the material is zero.) The force required to remove the residual magnetism from the material, is called the coercive force or coercivity of the material.

As the magnetising force is increased in the negative direction, the material will again become magnetically saturated but in the opposite direction (point “d”). Reducing **H** to zero brings the curve to point “e”. It will have a level of residual magnetism equal to that achieved in the other direction. Increasing **H** back in the positive direction will return **B** to zero. Notice that the curve did not return to the origin of the graph because some force is required to remove the residual magnetism. The curve will take a different path from point “f” back to the saturation point where it completes s the loop. The complete close loop *abcdefa* is called as a *hysteresis loop*. Hard magnetic materials have wider hysteresis loops as compared to that of soft magnetic materials as shown in figure 4.7.

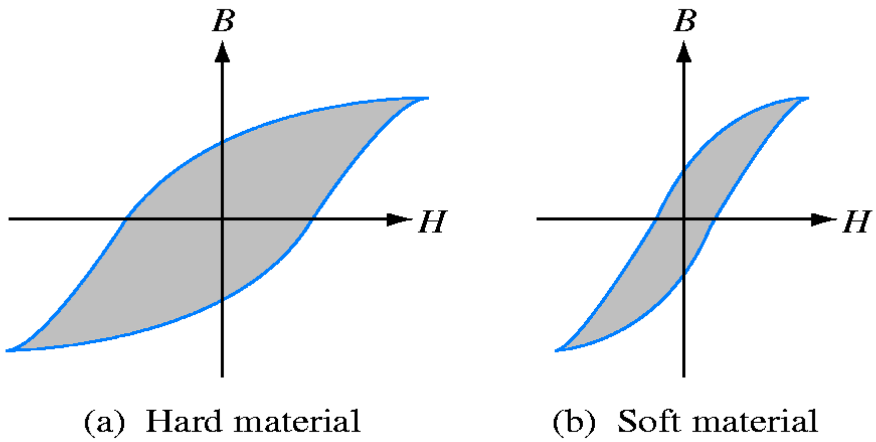
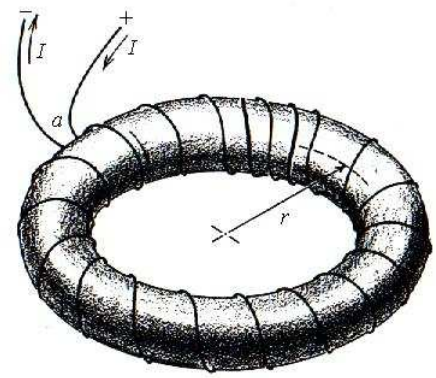


Figure 4.7: Hysteresis loops for (a) hard and (b) soft magnetic materials.

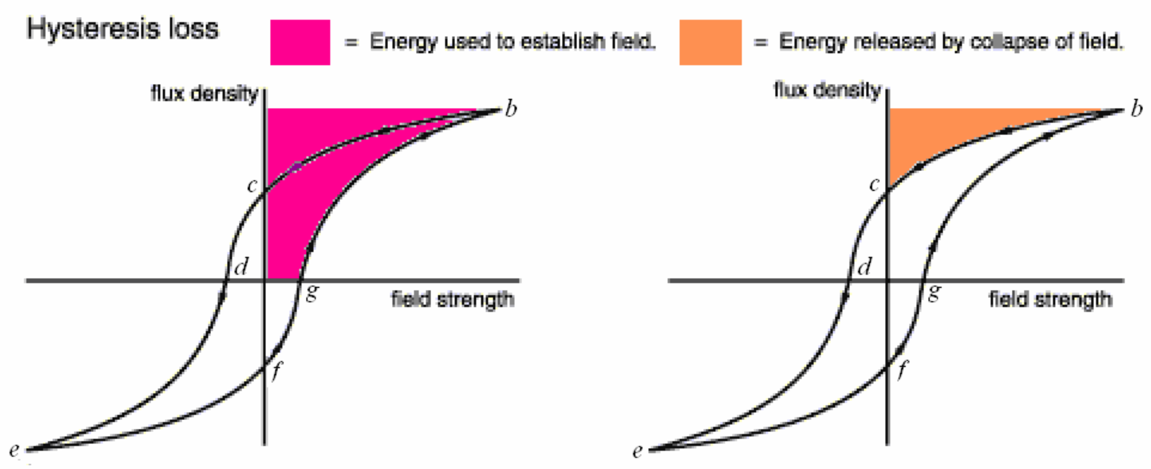
From the hysteresis loop, a number of primary magnetic properties of a material can be determined.

- *Retentivity* - it is a material's ability to retain a certain amount of residual magnetic field when the magnetising force is removed after achieving saturation. (The value of **B** at point “b” on the hysteresis curve).
- *Residual flux density* - the magnetic flux density that remains in a material when the magnetising force is zero. Note that residual flux density and retentivity are the same when the material has been magnetised to the saturation point. However, the level of *residual flux density* may be lower than the retentivity value when the magnetising force did not reach the saturation level.
- *Coercive force* - The amount of reverse magnetic field which must be applied to a magnetic material to make the magnetic flux return to zero. (The value of **H** at point “c” on the hysteresis curve).
- *Permeability*, μ - A property of a material that describes the ease with which a magnetic flux is established in the component.
- *Reluctance* - Is the opposition that a ferromagnetic material shows to the establishment of a magnetic field. Reluctance is analogous to the resistance in an electrical circuit.

Energy dissipated in a hysteresis loop



(a) A toroidal solenoid.



(b) Hysteresis loops.

Figure 4.8: (a) A solenoid and its (b) hysteresis loop.

Consider the solenoid that shown in figure 4.8(a), when the current I is increasing, the electromotive force (e.m.f) induced in the winding opposes the increase in current according to Lenz's law, the extra power spent by the source is

$$\frac{dW}{dt} = I \times \text{e.m.f.} = I\left(N\frac{d\Phi}{dt}\right) = NIA\frac{dB}{dt}, \tag{4.13}$$

where A is the cross sectional area of the core, N is the number of turns, B is the magnetic flux density induced in the core, and Φ is the magnetic flux generated. From the relation of $NI = HL$, Eq. (4.13) can be written as

$$\frac{dW}{dt} = NIA\frac{dB}{dt} = ALH\frac{dB}{dt} = VH\frac{dB}{dt}, \tag{4.14}$$

where L is the circumference of the core and $V = AL$ is the volume. Thus,

$$W_s = \int_g^b dW = V\int_g^b H\,dB\,, \qquad (4.15)$$

is the energy supplied by the source in going from the point "g" to the point "b" in figure 4.8(b). This integral corresponds to the shaded area as shown in figure 4.8(b) and is equal to the energy supplied per unit volume of the magnetic core.

When the current is in the same direction but is decreasing [figure 4.8(b)], the polarity of the induced e.m.f. is reversed, according to Lenz's law, with the result that the energy

$$W_r = V\int_b^c H\,dB\,, \qquad (4.16)$$

is returned to the source. Finally, the energy supplied by the source during one cycle is

$$W_T = V\oint H\,dB\,, \qquad (4.17)$$

where the integral is evaluated around the hysteresis loop (*gbcdefg*). The area of the hysteresis loop in tesla-ampere turns/meter or in weber-ampere turns/m^3 is therefore the number of joules dissipated per cubic meter and per cycle in the core (energy dissipated per unit volume per cycle). Sometime the energy dissipated per unit volume per cycle is also named as the *hysteresis loss*. This energy loss is due to the friction encountered during domain-wall motion and rotation.

Hysteresis loss can be minimised only by selecting a soft magnetic material with a narrow hysteresis loop as shown in figure 4.7(b). This is because the small loop area corresponds to a low hysteresis loss. Thus, soft magnetic materials are suitable for armature and transformer core. On the other hand, hard magnetic materials have larger loop areas [figure 4.7(a)] which correspond to higher hysteresis loss. Due to this reason, hard magnetic materials are not suitable for alternating magnetisation but they are suitable for permanent magnet.

Steinmetz hysteresis law

The power of the hysteresis loss is empirically given by Steinmetz hysteresis law,

$$P_h = fk_h B_p^x\,, \qquad \text{unit: W/m}^3 \qquad (4.18)$$

where f is the frequency of excitation, k_h is a constant determined by the nature of the magnetic material, B_p is the peak value of the magnetic flux density, and x is the Steinmetz coefficient ranging from 1.5 to 2.5.

4.3 Eddy Current Lost

When a changing magnetic field cuts through a sample of metal or magnetic materials that is not connected to a circuit, by Faraday's law, a circulating current is induced. This current is known as *eddy current*, it is localised within that material and has a flow pattern as shown in figure 4.9.

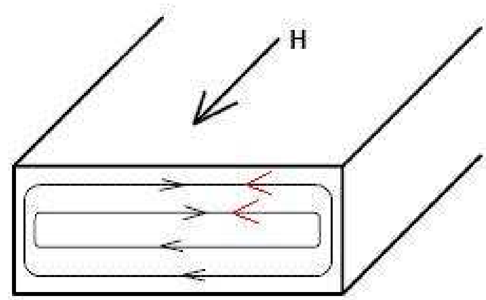


Figure 4.9: Eddy current (black arrows) induced within the metal or magnetic material and the current induced by the external magnetic field (red arrows).

This circulating current creates a magnetic field that opposes the external magnetic field. The direction of the eddy current is described by Lenz's law. The stronger of the external magnetic field or the greater of the electrical conductivity of the material, the eddy current that is developed will be stronger and also yields stronger opposing force.

Eddy current creates losses through Joule heating, and it reduces the efficiency of device that operates under alternating magnetic field condition such as iron core of transformers and alternating current motors. This power loss is known as eddy current loss due to the induced eddy current in the metal or magnetic materials.

In order to reduce the eddy current loss, the resistivity of the material is increased by adding silicon in the metal or ferromagnetic materials. Another effective way to achieve low eddy current loss is by using lamination of electrical metal sheets. These metal sheets are coated with insulator which breaks the eddy currents path as illustrated in the diagram below.

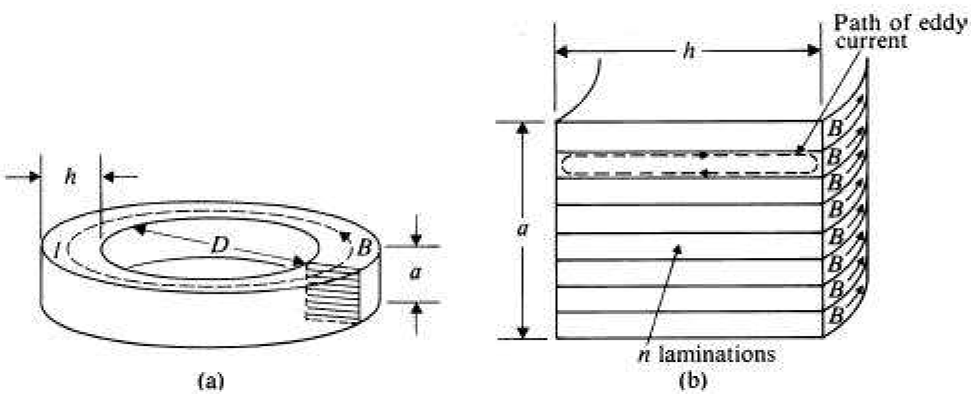


Figure 4.10: Eddy currents in a laminated toroidal core.

The power due to the eddy current loss is given as

$$P_e = k_e d^2 f^2 B_p^2, \qquad \text{unit: W/m}^3 \qquad (4.19)$$

where f is the frequency of excitation, k_e is a constant determined by the nature of the metal or magnetic material, B_p is the peak value of the magnetic flux density, and d is the thickness of the lamination. This formula is obtained under the assumption of global eddy current as shown in figure 4.9. This is incorrect for ferromagnetic materials due to the magnetic domains. When the excitation field varies, the domain walls move accordingly and local eddy currents are induced by the fluctuating of the local flux density caused by the domain wall motion as shown in figure 4.11.

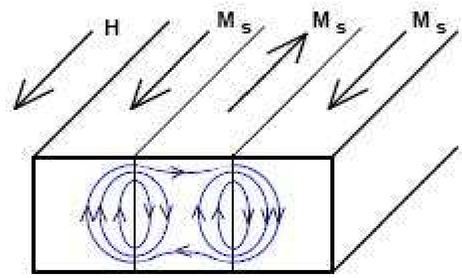


Figure 4.11: Eddy currents for domain wall model.

The total eddy current caused by the local eddy currents is generally higher than that predicted by the formulation under the global eddy current assumption. The different is known as the excess loss. Since it is very difficult to calculate the total average eddy current loss analytically, by statistical analysis, it was postulated that for most soft magnetic materials under a sinusoidal magnetic field excitation, the excess loss can be predicted by

$$P_{ex} = C_{ex} f^{3/2} B_p^{3/2}, \qquad \text{unit: W/kg} \qquad (4.20)$$

where f is the frequency of excitation, C_{ex} is a constant determined by the nature of the metal or magnetic material, and B_p is the peak value of the magnetic flux density. The total loss can be calculated by summing Eq. (4.18), Eq. (4.19), and Eq. (4.20)

$$P_{tot} = P_h + P_e + P_{ex}. \qquad (4.21)$$

Although eddy current reduces the efficiency of a electromagnetic device, but in a proper circumstances, it can be used as an non-destructive tool to measure material thickness and coating thickness. It also can be used in the crack detection, material identification, heat damage detection, and heat treatment monitoring.

4.4 The Concept of Magnetic Circuit

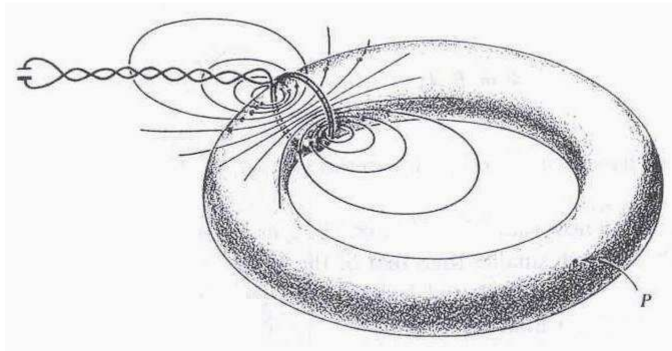


Figure 4.12: Ferromagnetic toroid with concentrated windings. The lines of force shown are in the plane of the toroid. They apply only when there is no iron present.

Figure 4.12 shows a ferromagnetic core around which is wound a coil of N turns carrying a current I. The magnetic flux Φ through some cross section of the core is to be determined. In the absence of ferromagnetic material, the lines of B are as shown in figure 4.12 and it is of the same order of magnitude at all points within the ferromagnetic material. This is because the magnetic induction due to the current I magnetises the core in the region near the coil, and the magnetisation produces Amperian currents that both increase B and extend it along the core. This further increases and extends the magnetisation and until the lines of B extend all around the core. Some of the lines of B escape into the air and then return to the core to pass again through the coil. This constitutes the leakage flux that may, or may not, be negligible. For example, if the toroid is made up of a long thin iron wire, the flux at P is negligible compared to that near the coil.

Suppose the cross section of the toroid is large enough to render the leakage flux. Then, applying Ampère's circuital law to a circular path of radius r going all around inside the toroid,

$$\oint \mathbf{H}\cdot d\mathbf{l} = NI,$$

$$2\pi r\frac{B}{\mu} = NI,$$

$$B = \frac{\mu NI}{2\pi r}. \tag{4.22}$$

Then, taking R_1 to be the radius corresponding to the average value of **B**, and R_2 to be the inner radius of the toroid, the flux through the core is

$$\Phi = \frac{\mu\pi R_2^2}{2\pi R_1}NI. \tag{4.23}$$

This equation shows that the magnetic flux is given by the *magnetomotance* or another name as *magneticmotive force* (m.m.f.), NI multiplied by the factor

$$\frac{\mu\pi R_2^2}{2\pi R_1}, \tag{4.24}$$

this is called the permeance of the magnetic circuit. The inverse of the permeance is called as the *reluctance*. Hence,

$$\text{Magnetic flux} = \text{Permeance} \times \text{Magnetomotive force} \tag{4.25}$$

$$= \text{Magnetomotive force/Reluctance.} \tag{4.26}$$

The above formula analogies with Ohm's law of electric circuit: if an electromotive force were induced in the core, the current would be

$$I = \frac{\sigma\pi R_2^2}{2\pi R_1}V\,. \tag{4.27}$$

Some corresponding quantities in electric and magnetic circuit are listed as below.

Electric quantities	**Magnetic quantities**
Current I	Magnetic flux Φ
Current density **J**	Magnetic flux density **B**
Conductivity σ	Permeability μ
Electromotive force = resistance $\times I$	Magnetomotive force = reluctance $\times \Phi$
Electric field intensity **E**	Magnetic field intensity **H**
Conductance = 1/resistance	Permeance = 1/reluctance
Resistance = $l/\sigma A$	Reluctance = $l/\mu A$

Table 4.2: Parallel quantities in electric and magnetic circuit.

The differences between electric and magnetic circuits are as below:

- The path of the magnetic flux flows is perpendicular to the current flows in the circuit. In other words, the directions of **B** and **J** are perpendicular.
- For a given temperature, electric resistance is constant and does not depend on current density. However, the magnetic reluctance depends on magnetic field and flux intensity since the permeability is not constant.
- Current flowing in a electric circuit involves dissipation of energy, but for magnetic circuit, energy is needed to generate magnetic flux.

4.4.1 Magnetic Circuit with and Air Gap

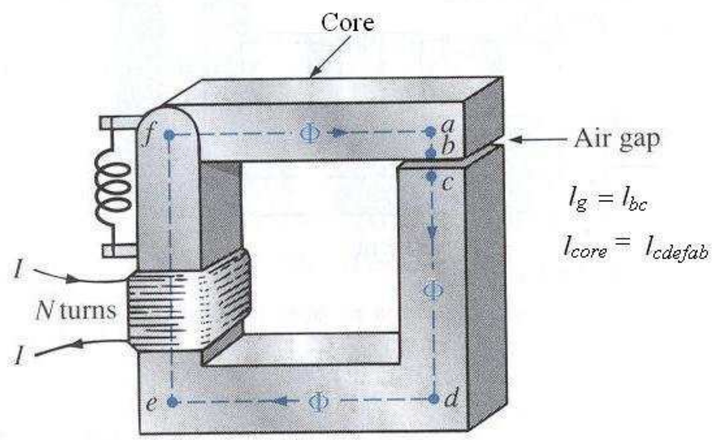
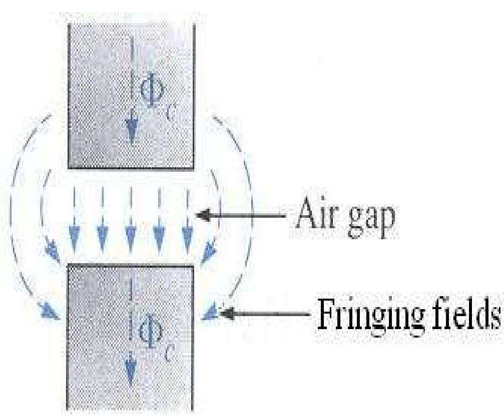
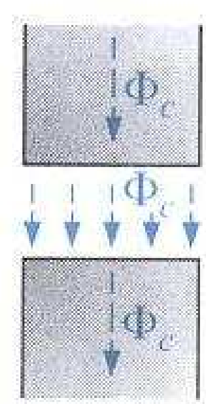


Figure 4.13: A simple magnetic circuit with an air gap.

Figure 4.13 shows a simple magnetic circuit with an air gap of length l_g cut in the middle of a leg. The winding provides *NI* ampere-turn. The spreading of the magnetic flux lines outside the common area of the core for the air gap is known as *fringing* field [figure 4.14(a)]. For simplicity, this effect is negligible and the flux distribution is assumed to be as in figure 4.14(b). It can be shown that the magnetic flux generated in the air gap is equal to the magnetomotive force *NI* divided by the sum of the reluctances of the core and of the air gap.



(a)



(b)

Figure 4.14: Air gaps (a) with fringing and (b) ideal.

Suppose the leakage flux is negligible. Applying the Ampère's circuital law, one gets

$$NI = H_c L_c + H_g L_g , \tag{4.28}$$

where the subscript *c* refers to the core and *g* to the air gap. The path length L_c in the core can be taken to be the length measured along the centre of the cross section of the core. H_c and H_g can be written in terms of the magnetic flux as

$$H_c = \frac{B_c}{\mu_c} = \frac{\Phi_c}{\mu_c A_c},$$
$$H_g = \frac{B_g}{\mu_g} = \frac{\Phi_g}{\mu_0 A_g}.$$

According to Gauss's law of magnetism, the net outward flux of **B** through any closed surface must be equal to zero. Hence, the flux of **B** must be the same over any cross section of the magnetic circuit and

$$\oint_s \mathbf{B} \cdot d\mathbf{s} = 0,$$
$$\Phi_c = \Phi_g = \Phi, \tag{4.29a}$$
$$B_c A_c = B_g A_g, \tag{4.29b}$$

where A_c and A_g are the cross sections of the core and of the air gap, respectively. Combining Eq. (4.28) and Eq. (4.29b) gives

$$B_g A_g \left[\frac{L_c}{\mu_c A_c} + \frac{L_g}{\mu_0 A_g} \right] = NI, \tag{4.30}$$

and the magnetic flux is

$$\Phi = B_g A_g = \frac{NI}{\dfrac{L_c}{\mu_c A_c} + \dfrac{L_g}{\mu_0 A_g}}. \tag{4.31}$$

The denominator of Eq. (4.31) gives the reluctances of the core and air gap in series. Since the leakage flux is neglected, this equation gives the upper limit for Φ.

<u>Magnetic circuit equivalent and electric circuit analogy for the electromagnet circuit</u>

If the reluctance of core and air gap is written as $R_c = \dfrac{L_c}{\mu_c A_c}$ and $R_g = \dfrac{L_g}{\mu_0 A_g}$, respectively, Eq. (4.31) can be rewritten as

$$NI = (R_c + R_g)\Phi,$$
$$F = (R_c + R_g)\Phi. \tag{4.32}$$

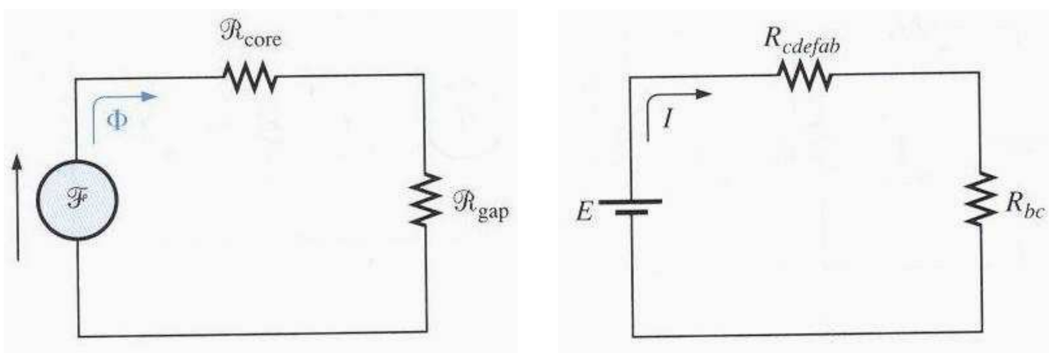
The above magnetic circuit with an air gap can be represented in a magnetic circuit diagram as shown in figure 4.15(a) and it is analogous to a series electric circuit in figure 4.15(b). Further if H_cL_c and H_gL_g are regarded as the m.m.f. drops (analogy to voltage drops in electric circuit) across the reluctance of the core and air gap respectively, Eq. (4.32) derived from Ampère's circuital law can be interpreted as an analogue to the *Kirchhoff's voltage law* (KVL) in electric circuit theory,

$$\sum_k V_k = 0,$$

$$\sum_k F_k - \sum_k R_k \Phi_k = 0.$$

$$\sum_k F_k = \sum_k R_k \Phi_k. \tag{4.33}$$

Eq. (4.33) states that the algebraic sum of the rises and drops of the magnetomotive force around a closed loop of a magnetic circuit is equal to zero. In other words, the sum of the magnetomotive force rises equals the sum of the magnetomotive drops around a closed loop.



(a) (b)

Figure 4.15: (a) Magnetic circuit equivalent and (b) electric circuit analogy for the electromagnet.

If Φ_c and Φ_g are regarded as the "current entering/leaving" a junction in the magnetic circuit, Eq. (4.29a) derived Gauss's law of magnetism can be interpreted as an analogue to the *Kirchhoff's current law* (KCL) in electric circuit theory,

$$\sum I_{entering} = \sum I_{leaving},$$

$$\sum \Phi_{entering} = \sum \Phi_{leaving}. \tag{4.34}$$

Eq. (4.34) states that the algebraic sum of the fluxes entering or leaving a junction of a magnetic circuit is equal to zero. In other words, the sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction.

4.4.2 Magnetic Circuit with a Permanent Magnet

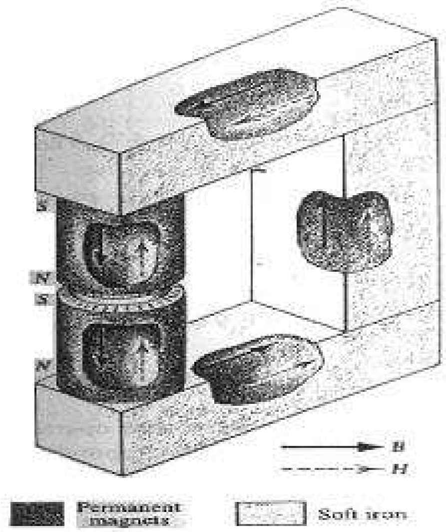


Figure 4.16: Magnetic circuit with a pair of permanent magnets.

Figure 4.16 shows a magnetic circuit with a pair of permanent magnets. The directions of **B** and **H** vectors in different parts of the magnetic circuit are shown in the figure. In the soft iron core, **B** and **H** are in the same direction and the operating point on the hysteresis curve is situated somewhere in the first quadrant. In the air gap, **B** and **H** are also oriented in the same direction. However, **B** and **H** are oriented in opposite direction for permanent magnet since the operating point for the permanent magnet is in the second quadrant on the hysteresis curve.

To calculate the magnetic flux generated within the air gap, for simplicity, we assume that the leakage flux is negligible. Since there are neither free currents nor displacement currents, the line integral of **H·dl** around the magnetic circuit must be zero. Then,

$$H_i L_i + H_g L_g - H_m L_m = 0\,, \tag{4.35}$$

$$H_i L_i + H_g L_g = H_m L_m\,, \tag{4.36}$$

where the subscripts *i*, *g*, and *m* refer to the soft iron core, the air gap, and the permanent magnet, respectively. L_i, L_g, and L_m are the Amperian paths for iron core, air gap, and permanent magnet respectively and the total length around the circuit is $L_i + L_g + L_m$. H_i, H_g, and H_m can be written in terms of the magnetic flux as

$$H_i = \frac{B_i}{\mu_i} = \frac{\Phi_i}{\mu_i A_i}\,,$$

$$H_g = \frac{B_g}{\mu_g} = \frac{\Phi_g}{\mu_0 A_g}\,,$$

$$H_m = \frac{B_m}{\mu_m} = \frac{\Phi_m}{\mu_m A_m}\,.$$

Again, Φ is the same over any cross section, thus

$$\Phi_i = \Phi_g = \Phi_m = \Phi, \tag{4.37}$$

$$A_i B_i = A_g B_g = A_m B_m . \tag{4.38}$$

where A_i, A_g, and A_m are the cross sections of the iron core, air gap, and permanent magnet, respectively. Combining Eq. (4.36) and (4.38) gives

$$B_g A_g \left[\frac{L_i}{\mu_i A_i} + \frac{L_g}{\mu_0 A_g} \right] = H_m L_m , \tag{4.39}$$

and the magnetic flux is

$$\Phi = B_g A_g = \frac{H_m L_m}{\dfrac{L_i}{\mu_i A_i} + \dfrac{L_g}{\mu_0 A_g}} . \tag{4.40}$$

The denominator of Eq. (4.40) gives the reluctances of the iron core and of the air gap in series. This equation is similar to Eq. (4.31) except that NI is replaced by $H_m L_m$. This shows that permanent magnet could be used to replace the current source to establish the magnetic flux in the circuit. Since the leakage flux is neglected, this equation gives the upper limit for Φ.

If the reluctance of the iron core and the air gap is written as $R_i = \dfrac{L_i}{\mu_i A_i}$ and $R_g = \dfrac{L_g}{\mu_0 A_g}$, respectively, Eq. (4.40) can be rewritten as

$$H_m L_m = (R_i + R_g)\Phi . \tag{4.41}$$

The design of a magnetic circuit energised by permanent magnet is complicated due to the position of the operating point (the relation between **B** and **H**) on the hysteresis curve. Thus, one usually uses an operating point that makes the *energy product* $H_m B_m$ maximum. The application of a magnetic circuit is to produce a magnetic field in an air gap, and the magnetic energy required is the volume of the air gap, $A_g L_g$ multiplied by the energy density $\dfrac{B_g^2}{2\mu_0}$. That is,

$$\frac{1}{2\mu_0} A_g L_g B_g^2 = \frac{1}{2}(H_g L_g)(A_g B_g). \tag{4.42}$$

If the reluctance of the iron core is much smaller than the air gap, the m.m.f. H_iL_i can be neglected. Thus, from Eq. (4.36) and (4.38), Eq. (4.42) is written as

$$\frac{1}{2\mu_0} A_g L_g B_g^2 = \frac{1}{2}(H_m L_m)(A_m B_m),$$
$$= \frac{1}{2}(H_m B_m)(A_m L_m). \tag{4.43}$$

The magnetic energy in the air gap is therefore equal to one-half of the energy product multiplied by the volume of the permanent magnet. As a rule, one requires that the volume A_mL_m be as small as possible, for reasons of economy, size, and weight. Then the operating point on the hysteresis curve is chosen so that the energy product H_mB_m is maximum.

<u>Attractive force between two iron surfaces separated by an air gap</u>

From Eq. (4.42), the magnetic energy density stored in the air gap between the two surfaces of the iron cores is

$$W_0 = \frac{B_g^2}{2\mu_0}. \tag{4.44}$$

If one end of the iron cores is made to be movable and pulled away from the other end of the iron core by a force P N for a distance Δd meters. The additional magnetic energy stored in the additional volume of the air gap ($A_g\Delta d$) is caused by the work done by the force. Thus, one gets

$$\Delta W = P\Delta d = W_0\Delta V = \frac{B_g^2}{2\mu_0}\left(A_g \Delta d\right). \tag{4.45}$$

This indicates that the magnetic energy stored in the air gap can be increased by increases the volume of the air gap. The force required to increase the additional magnetic energy is then given as

$$P = \frac{B_g^2 A_g}{2\mu_0}. \tag{4.46}$$

Some examples of magnetic circuit calculation

Example 4.1

Find the value of I required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-4}$ Wb in the series magnetic circuit as shown in figure 4.17. Calculate the force exerted on the armature (moving part) when the flux is established. The relative permeability for the steel is $\mu_r = 1424$.

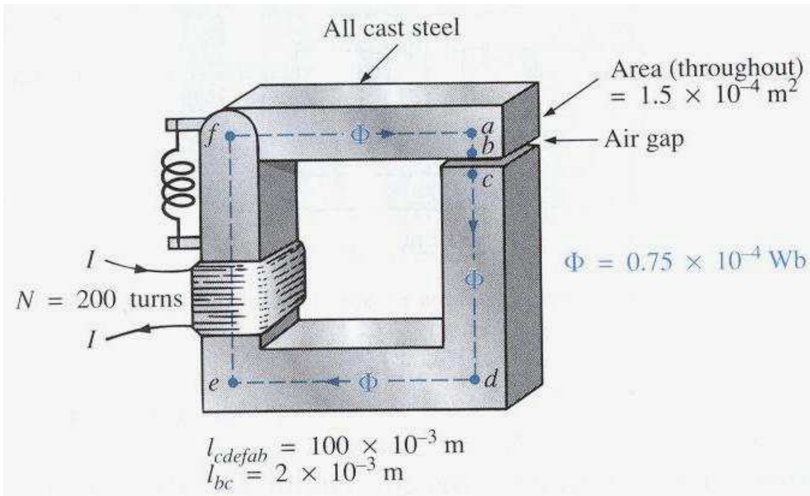


Figure 4.17: Magnetic circuit with an air gap.

Solution:

The above device can be analysed by its magnetic circuit equivalent and its electric circuit analogy as shown in figure 4.18.

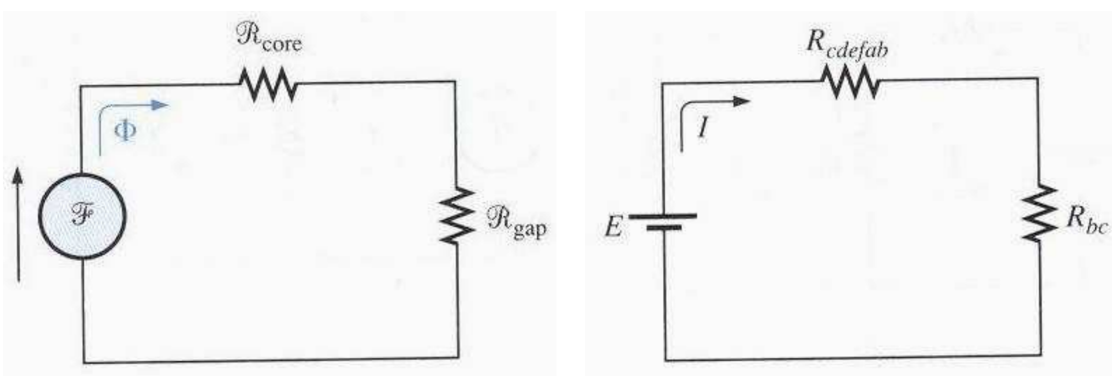


Figure 4.18: (a) Magnetic circuit equivalent and (b) electric circuit analogy for the electromagnetic device in figure 4.17.

From the Gauss law (analogy to KCL in electric circuit), the flux density for each section is

$$B = \frac{\Phi}{A} = \frac{0.75\times10^{-4}\ \text{Wb}}{1.5\times10^{-4}\ \text{m}^2} = 0.5\,\text{T}.$$

The magnetic field intensity of steel is

$$H_c = \frac{B}{\mu_c} = \frac{B}{\mu_r\mu_0} = \frac{0.5\,\text{T}}{1424\times4\pi\times10^{-7}\ \text{Hm}^{-1}} = 279\ \text{At/m.}$$

The magnetic field intensity of air gap is

$$H_g = \frac{B}{\mu_g} = \frac{B}{\mu_0} = \frac{0.5\,\text{T}}{1\times4\pi\times10^{-7}\ \text{Hm}^{-1}} = 3.98\times10^5\ \text{At/m.}$$

The magnetomotive force drops are

$$H_c l_c = (279\ \text{At/m})(100\times10^{-3}\ \text{m}) = 28\ \text{At.}$$
$$H_g l_g = (3.98\times10^5\ \text{At/m})(2\times10^{-3}\ \text{m}) = 796\ \text{At.}$$

Applying Ampère circuital law (analogy to KVL in electric circuit),

$$\begin{aligned} NI &= H_c l_c + H_g l_g \\ &= 28\ \text{At} + 796\ \text{At} \\ (200\ \text{t})I &= 824\ \text{At} \\ I &= 4.12\ \text{A.} \end{aligned}$$

Energy stored in the air gap, W = Volume of air gap, V_g × magnetic energy density, W_0

$$= A_g l_g \times \frac{B_g^2}{2\mu_0}.$$

W = mechanical energy to close the air gap
= Force, P × length of the air gap, l_g

$$A_g l_g \times \frac{B_g^2}{2\mu_0} = P l_g$$

$$\begin{aligned} P &= \frac{A_g B_g^2}{2\mu_0} \\ &= \frac{1.5\times10^{-4}\ \text{m}^2 \times (0.5\,\text{T})^2}{2\times4\pi\times10^{-7}\ \text{Hm}^{-1}} \\ &= 14.92\ \text{N} \end{aligned}$$

Example 4.2

Determine the value of I required to establish a magnetic flux of $\Phi = 1.54 \times 10^{-4}$ Wb in the section of the core indicated in figure 4.19. The relative permeability for the steel at region *bcde*, *be*, and *efab* are $\mu_2 = 4972$, $\mu_1 = 4821$, and $\mu_T = 2426$, respectively.

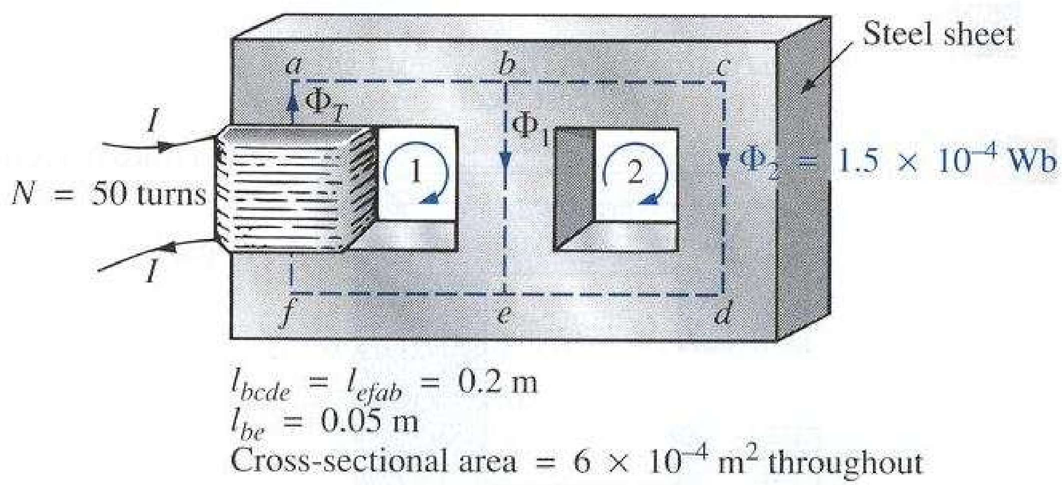
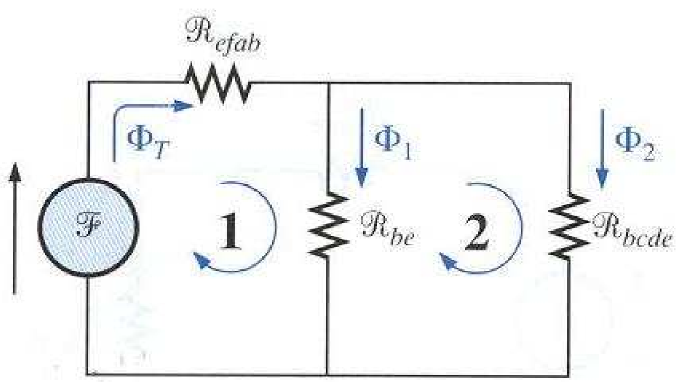


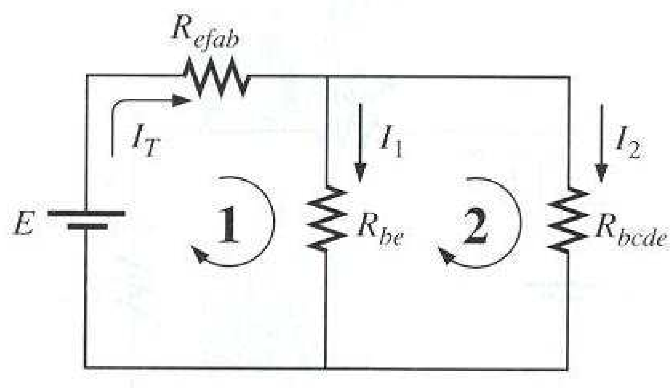
Figure 4.19: Electromagnetic device in a series-parallel configuration.

Solution:

The above device can be analysed by its magnetic circuit equivalent and its electric circuit analogy as shown in figure 4.20.



(a)



(b)

Figure 4.20: (a) Magnetic circuit equivalent and (b) electric circuit analogy for the electromagnetic device in example 4.19.

For loop 2, the magnetic flux density is

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5\times10^{-4}\ \text{Wb}}{6\times10^{-4}\ \text{m}^2} = 0.25\,\text{T},$$

and the corresponding magnetic field intensity is

$$H_{bcde} = \frac{B_2}{\mu_{bcde}} = \frac{B_2}{\mu_2\mu_0} = \frac{0.25\,\text{T}}{4972\times4\pi\times10^{-7}\ \text{Hm}^{-1}} = 40\ \text{At/m}.$$

Applying Ampère circuital law around loop 2 of figure 4.19,

$$\begin{aligned} H_{be}l_{be} - H_{bcde}l_{bcde} &= 0 \\ H_{be}(0.05\,\text{m}) - (40\ \text{At/m})(0.2\,\text{m}) &= 0 \\ H_{be} &= \frac{8\ \text{At}}{0.05\,\text{m}} \\ &= 160\ \text{At/m}. \end{aligned}$$

Then, the magnetic flux density in region ‘be’ is

$$B_1 = B_{be} = \mu_{be}H_{be} = \mu_1\mu_0H_{be} = 4821\times4\pi\times10^{-7}\ \text{H/m}\times160\ \text{At/m} = 0.97\,\text{T}.$$

The flux in this region is

$$\Phi_1 = B_1A = (0.97\,\text{T})(6\times10^{-4}\ \text{m}^2) = 5.82\times10^{-4}\ \text{Wb}.$$

For loop 1, apply Gauss law (analogy to KCL in electric circuit) gives

$$\Phi_T = \Phi_1 + \Phi_2 = (5.82 + 1.5) \times 10^{-4} \text{ Wb}$$
$$= 7.32 \times 10^{-4} \text{ Wb}$$
$$\mathrm{B} = \frac{\Phi_T}{A} = \frac{7.32 \times 10^{-4} \text{ Wb}}{6 \times 10^{-4} \text{ m}^2} = 1.22 \text{ T}.$$

Then,

$$H_{efab} = \frac{B}{\mu_{efab}} = \frac{B}{\mu_T \mu_0} = \frac{1.22 \text{ T}}{2426 \times 4\pi \times 10^{-7} \text{ Hm}^{-1}} = 400 \text{ At/m}.$$

Applying Ampère circuital law (analogy to KVL in electric circuit),

$$NI = H_{efab} l_{efab} + H_{be} l_{be}$$
$$NI = (400 \text{ At/m})(0.2 \text{ m}) + (160 \text{ At/m})(0.05 \text{ m})$$
$$(50 \text{ t})I = 80 \text{ At} + 8 \text{ At}$$
$$I = 1.76 \text{ A}.$$

<u>Example 4.3</u>
The core of figure 4.21 is made of cast steel. Calculate the current *I* that needed to establish a flux of $\Phi_g = 6 \times 10^{-3}$ Wb at the air gap if fringing field is neglected.

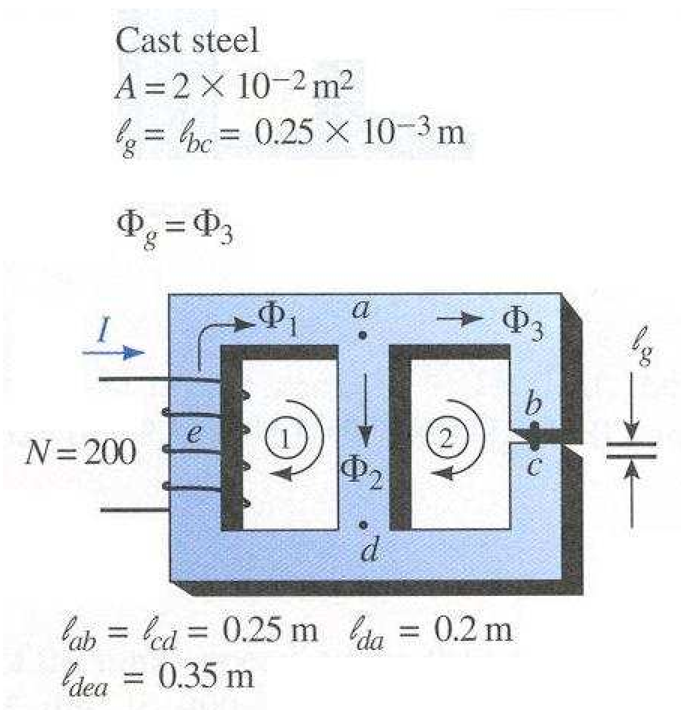


Figure 4.21: Series-parallel magnetic circuit made of cast steel core.

Solution:

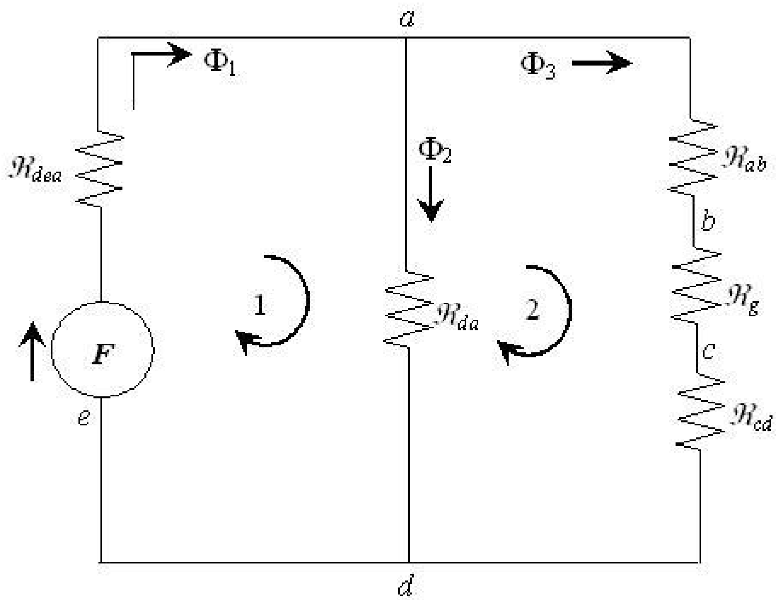


Figure 4.22(a): Magnetic circuit equivalent.

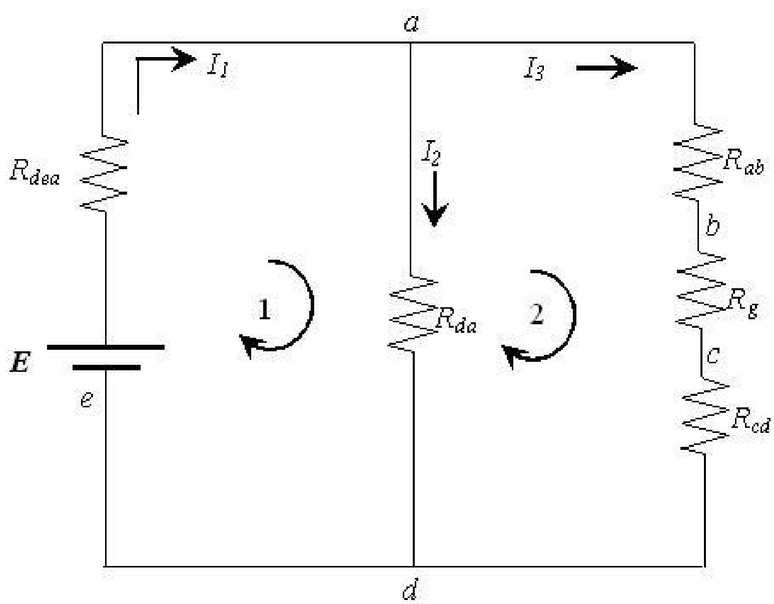


Figure 4.22(b): Electric circuit analogy.

Consider each section in turn.

For air gap,

$$B_g = \Phi_g/A_g = 6 \times 10^{-3}\ \text{Wb}/2 \times 10^{-2}\ \text{m}^2 = 0.3\ \text{T}.$$
$$H_g = B_g/\mu_0 = 0.3\ \text{T}/(4\pi \times 10^{-7}\ \text{H/m}) = 2.388 \times 10^5\ \text{At/m}.$$

For section *ab* and *cd*,

$B_{ab} = B_{cd} = B_g = 0.3$ T.
$H_{ab} = H_{cd} = 250$ At/m. (refer to *B*-*H* curve in figure 4.26).

Apply Ampère circuital law (KVL) at loop 2,

Since the flux in leg *da* flows in the opposite direction of the flux in leg *ab*, leg *cd*, and air gap, the corresponding term of $H_{da}l_{da}$ will be subtractive. Also, $NI = 0$ for loop 2. Thus,

$$\begin{aligned}
\sum_{loop2} NI &= \sum_{loop2} Hl \\
0 &= H_{ab}l_{ab} + H_g l_g + H_{cd}l_{cd} - H_{da}l_{da} \\
&= (250)(0.25) + \left(2.388\times10^5\right)\left(0.25\times10^{-3}\right) + (250)(0.25) - 0.2H_{da} \\
&= 62.5 + 59.7 + 62.5 - 0.2H_{da} \\
&= 184.7 - 0.2H_{da}
\end{aligned}$$

Thus,

$H_{da} = 925$ At/m.
$B_{da} = 1.12$ T (from figure 4.26).

$\Phi_2 = B_{da}A = 1.12 \times 0.02 = 2.24 \times 10^{-2}$ Wb.
$\Phi_1 = \Phi_2 + \Phi_3 = 2.84 \times 10^{-2}$ Wb.
$B_{dea} = \Phi_1/A = (2.84 \times 10^{-2})/0.02 = 1.42$ T.
$H_{dea} = 2125$ At/m (from figure 4.26).

Apply Ampère circuital law at loop 1,

$NI = H_{dea}l_{dea} + H_{da}l_{da} = (2125)(0.35) + 184.7 = 929$ At
$I = 929/200 = 4.65$ A.

Example 4.4

The core of the magnetic device as shown in figure 4.23 is made of cast-iron and it is symmetrical both left and right arms. Find the current *I* that needed to establish flux density of 30 μWb at the right arm of the core.

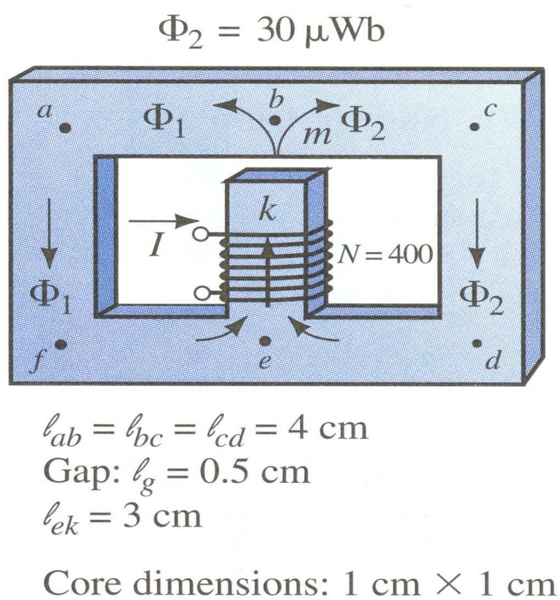


Figure 4.23: Series-parallel magnetic circuit made of cast iron core.

Solution:

The magnetic circuit equivalent and its electric circuit analogy are shown in below.

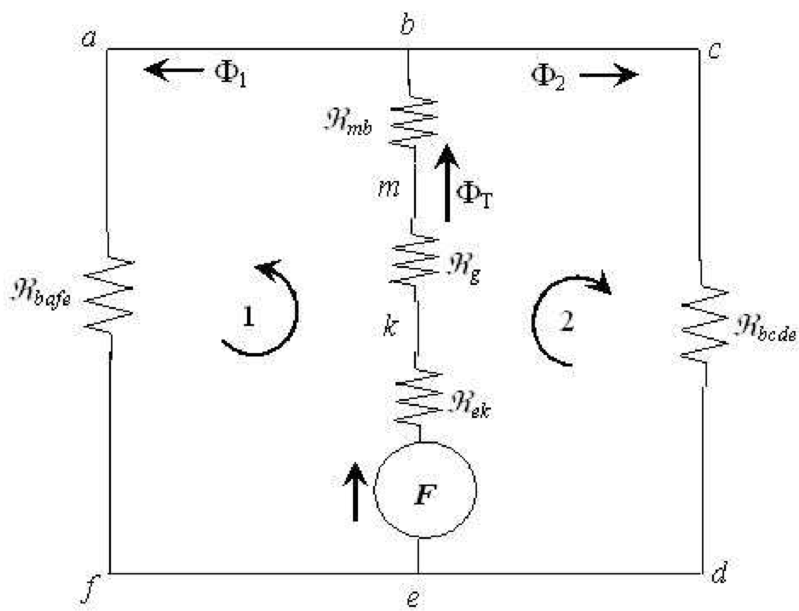


Figure 4.24(a): Magnetic circuit equivalent.

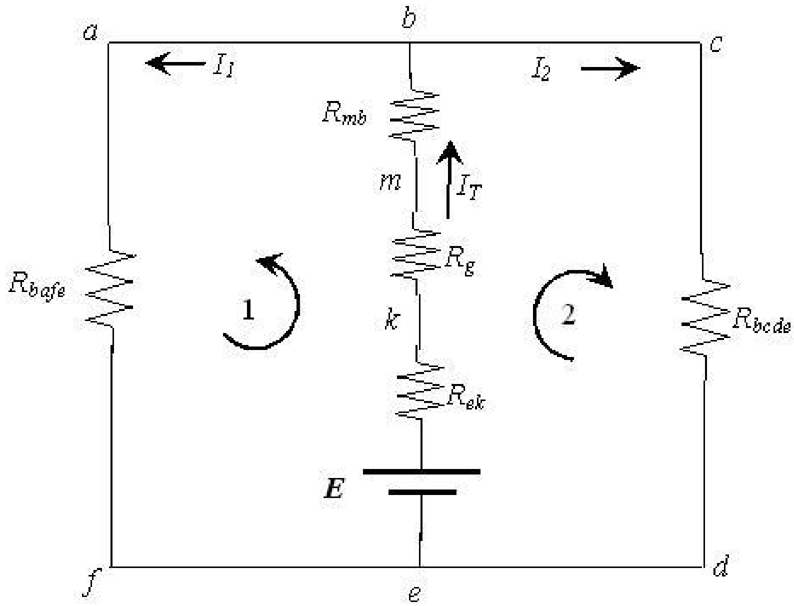


Figure 4.24(b): Electric circuit analogy.

Since the magnetic device is symmetry and made by the same material at both left and right arms, we can concentrate the analysis of the circuit on either loop 1 or loop 2.

From Gauss law, one gets

$$\Phi_1 = \Phi_2.$$
$$\Phi_T = \Phi_1 + \Phi_2.$$
$$= 2\Phi_2.$$

Apply Ampère circuital law at left hand arm,

$$\sum\nolimits_{loop1} NI = \sum\nolimits_{loop1} Hl$$

$$NI = H_{ek}l_{ek} + H_g l_g + H_{mb}l_{mb} + H_{bafe}l_{bafe}$$

$$NI = H_{ek}l_{ek} + H_g l_g + H_{mb}l_{mb} + H_{bafe}\left(l_{ba} + l_{af} + l_{fe}\right) \tag{1}$$

$$B_{ek} = \frac{\Phi_T}{A} = \frac{2\Phi_2}{A} = \frac{2\times 30\times 10^{-6}\ \text{Wb}}{(0.01\times 0.01)\,\text{m}^2} = 0.6\,\text{T}$$

$$H_{ek} = H_{mb} = 2600\ \text{At/m} \qquad \text{(from figure 4.26)} \tag{2}$$

$$H_g = \frac{B_g}{\mu_0} = \frac{\Phi_T}{\mu_0 A} = \frac{2\Phi_2}{\mu_0 A} = \frac{2\times 30\times 10^{-6}\ \text{Wb}}{\left(4\pi\times 10^{-7}\ \text{H/m}\right)(0.01\times 0.01)\,\text{m}^2} = 4.77\times 10^5\ \text{At/m.} \tag{3}$$

$$B_{bafe} = \frac{\Phi_1}{A} = \frac{30\times 10^{-6}\ \text{Wb}}{(0.01\times 0.01)\,\text{m}^2} = 0.3\,\text{T}$$

$$H_{bafe} = 750\ \text{At/m} \qquad \text{(from figure 4.26)} \tag{4}$$

Substitute (2), (3), and (4) in (1) yields

$$
\begin{aligned}
I &= \frac{1}{N}\left[\begin{array}{l}(2600\ \text{At/m}\times 0.03\ \text{m}) + (4.77\times 10^{5}\ \text{At/m}\times 0.5\times 10^{-2}\ \text{m}) \\ +(2600\ \text{At/m}\times 0.5\times 10^{-2}\ \text{m}) + (750\ \text{At/m}\times 3\times 0.04\ \text{m})\end{array}\right] \\
&= \frac{2566}{400} \\
&= 6.415\ \text{A.}
\end{aligned}
$$

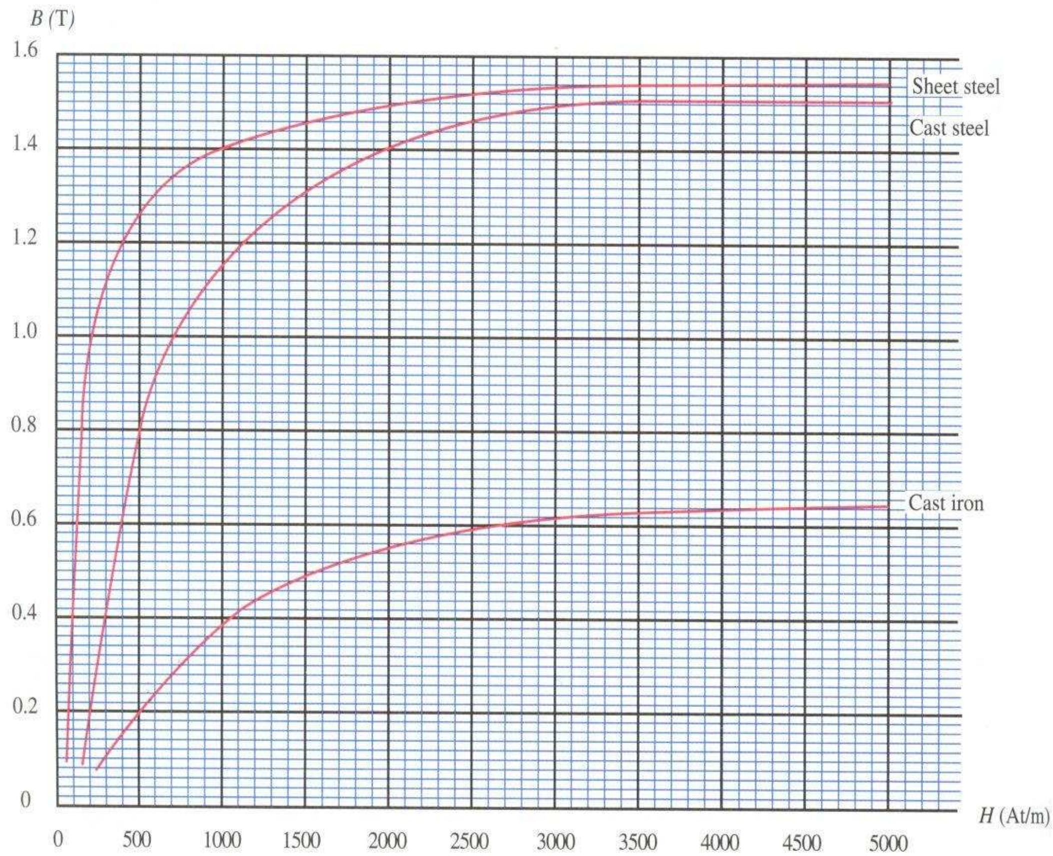


Figure 4.26: *B-H* curves for selected materials.

4.5 Application – Maglev Train

Magnetic levitation train, Maglev, is a system in which the train runs levitated from the guideway (track) by using electromagnetic forces between superconducting magnets on board the train and coils on the guideway. The acceleration and braking system of the train are also handled by using electromagnets. The frictionless motion offered by magnetic levitation provides an extremely smooth, quite, and fast ride with speeds in excess of 500 km/h. Maglev is considered as a relatively cheap and fast transportation alternative to conventional rail trains, cars or airplanes.

The world's first commercial application of the high-speed Maglev system was implemented in Shanghai, China in 2002 and named as Shanghai Maglev Train. This system runs from downtown Shanghai to the new Shanghai airport at Pudong over a track length of 30 km. It was constructed by a German company called Transrapid International.

Figure 4.27: Shanghai maglev train.

<u>Operation Principle of Maglev</u>

Principle of magnetic levitation

The "8" figured levitation coils are installed on the sidewalls of the guideway. When the on-board superconducting magnets pass at a high speed about several centimetres below the centre of these coils, an electric current is induced within the coils, which then act as electromagnets temporarily. As a result, there are forces which push the superconducting magnet upwards and ones which pull them upwards simultaneously, thereby levitating the Maglev train.

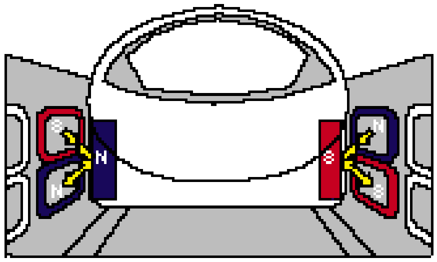


Figure 4.28(a): Magnetic levitation.

Principle of lateral guidance

The levitation coils (magnets) are connected on both sides of the guideway. In a running Maglev train, a repelling force between the train and the guideway on one side (N repels N), and an attracting force on the other side of the train between the train and the guideway (N attracts S), keeps the train at the center of the guideway.

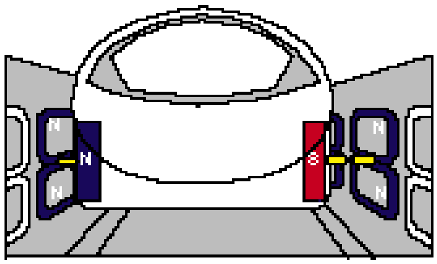


Figure 4.28(b): Magnetic guidance.

Principle of propulsion

A repelling force and an attracting force created between the magnets on the train and in the guideway are used to propel the vehicle forward. The magnetic coils located on the sidewalls on both sides of the guideway are powered by an alternating current from the station, creating a shifting magnetic field on the guideway. The superconducting magnets on the train are attracted and pushed by the shifting field in the guideway, propelling the Maglev train.

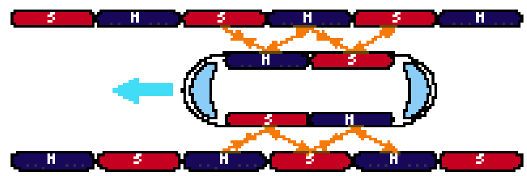


Figure 4.28(c): Magnetic propulsion.

Material taken from:
http://www.newton.mec.edu/Brown/TE/MAGLEV/RTRI/maglev_principle_E.html

Extra readings on Maglev:

1. Video clip about the operation principle of Maglev by Transrapid International.
2. Video clip of Maglev in Shanghai by Transrapid International.
   URL: http://users.bigpond.net.au/maglevvideogallery/
3. Maglev Train: http://en.wikipedia.org/wiki/Maglev

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2. Stuart M. Wentworth, Fundamentals of Electromagnetics with Engineering Applications, John Wiley & Sons Inc., U.S., 2005.
3. William H. Hayt, Jr. and John A. Buck, Engineering Electromagnectics, 7^{th} Ed., McGraw-Hill, 2006.
4. Matthew N. O. Sadiku, Elements of Electromagnetics, 3^{rd} Ed., Oxford University Press Inc., New York, 2003.
5. Robert L. Boylestad, Introductory Circuit Analysis, 8^{th} Ed., Prentice Hall International Inc., Upper Saddle River, New Jersey, 1997.

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